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On FGSPR Compact Spaces

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Abstract: *- A new type of fuzzy compact space and fuzzy compact function namely fgspr compact space and fgspr compact function with the concept of fgspr-open set are introduced. Some characterizations on their properties are obtained.*
Keywords: *- fgspr compact space and fgspr compact function*

I. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [10]. Subsequently, several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. Fuzzy topology and fuzzy compactness was first introduced by Chang [2]. The concept of fgspr-closed set and fgspr-open set in fuzzy topological space was introduced by M. Thiruchelvi and Gnanambal [6]. In this paper fgspr compact space and fgspr compact function are introduced and their properties are studied.

II. PRELIMINARIES

Let X, Y and Z be fuzzy sets. Throughout this paper $(X,\tau),(Y,\sigma)$ and (Z,η) (or simply X, Y and Z) represent nonempty fuzzy topological spaces on which no separation axioms are assumed unless otherwise stated. Let $f:(X,\tau)\to(Y,\sigma)$ be a function from a fuzzy topological space X to fuzzy topological space Y. Let us recall the following definitions which we requires later as follows.

Definition 2.1: A fuzzy set λ in a fuzzy topological space (X,τ) is called
(1) a fuzzy semi-preopen set [8] if $\lambda\le cl(int(cl(\lambda)))$ and a fuzzy semi-preclosed set if $int(cl(int(\lambda)))\le\lambda$.
(2) a fuzzy regular open set [1] if $int(cl(\lambda))=\lambda$ and a fuzzy regular closed set if $cl(int(\lambda))=\lambda$.

Definition 2.2: A fuzzy set λ in a fuzzy topological space (X,τ) is called
(1) a fuzzy generalized semi preregular closed set (briefly, fgspr-closed) [6] if $spcl(\lambda)\le\mu$, whenever $\lambda\le\mu$ and μ is a fuzzy regular open set in X.
(2) a fuzzy generalized semi preregular open set (briefly, fgspr-open) [6] if $\mu\le spint(\lambda)$, whenever $\mu\le\lambda$ and μ is a fuzzy regular closed set in X.

Definition 2.3: Let X and Y be two fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\sigma)$ is called
(1) a fuzzy generalized semi preregular continuous (briefly, fgspr-continuous) [5] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.
(2) a fuzzy generalized semi preregular irresolute (briefly, fgspr-irresolute) [5] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set λ in Y.
(3) a fuzzy open (briefly, f-open) [9] if $f(\lambda)$ is a fuzzy open set in Y, for every fuzzy open set λ in X.
(4) a fuzzy generalized semi preregular open (briefly, fgspr-open) [7] if $f(\lambda)$ is a fuzzy generalized semi preregular open set in Y, for every fuzzy open set λ in X.

Definition 2.4: A fuzzy topological space (X,τ) is called
(1) a fuzzy semi preregular $T_{1/2}$ space [5] if every fgspr-closed is fuzzy semi preclosed.
(2) a fuzzy semi preregular $T^{*}_{1/2}$ space [5] if every fgspr-closed is fuzzy closed.

Definition 2.5: A fuzzy topological space (X, τ) is called
(1) a fuzzy compact space (briefly, f-compact) [2] if every fuzzy open cover of X has a finite sub cover.
(2) a fuzzy semi pre compact space (briefly, fsp-compact) [4] if every fuzzy semi pre open cover of X has a finite sub cover.

Definition 2.6: Let X and Y be two fuzzy topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a fuzzy compact (briefly, f-compact) [3] if $f^{-1}(\lambda)$ is a fuzzy compact set in X, for every fuzzy compact set λ in Y.

III. FGSPR COMPACT SPACE

In this section, a new type of fuzzy compact space called fuzzy generalized semi preregular compact space is defined and its properties are studied.

Definition 3.1: A collection $\{\lambda_i : i \in I\}$ of fgspr-open sets in a fuzzy topological space (X, τ) is called fgspr-open cover of a fuzzy set μ in X if $\mu \leq \vee\{\lambda_i : i \in I\}$.

Definition 3.2: A fuzzy set λ in (X, τ) is said to be fgspr compact relative to X if every collection $\{\lambda_i : i \in I\}$ of fgspr-open sets of X such that $\lambda \leq \vee\{\lambda_i : i \in I\}$ there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $\lambda \leq \vee_{j=1}^{n} \{\lambda_{i_j} : i \in I\}$.

Example 3.3: Let X = {a, b, c} and consider the fuzzy sets $\lambda_1 = \{(a, 0.7), (b, 0.5), (c, 0.2)\}$, $\lambda_2 = \{(a, 0.8), (b, 0.5), (c, 0.3)\}$, $\lambda_3 = \{(a, 0), (b, 0.4), (c, 0.2)\}$ and $\lambda_4 = \{(a, 0.5), (b, 0.4), (c, 0.1)\}$. Let $\tau = \{0, \lambda_1, 1\}$. The collection of fgspr-open sets $\lambda = \{0, \lambda_2, \lambda_3, 1\}$ such that $\lambda_4 \leq \vee\lambda$ and the sub collection of fgspr-open sets $\mu = \{0, \lambda_2, 1\}$ such that $\lambda_4 \leq \vee\mu$. Hence λ_4 is fgspr compact relative to X.

Definition 3.4: A fuzzy set λ in (X, τ) is said to be fgspr compact if λ is fgspr compact relative to X.

Definition 3.5: A fuzzy topological space (X, τ) is called fgspr compact if every fgspr-open cover of X has a finite sub cover.

Example 3.6: Let X = {a, b, c} and consider the fuzzy sets $\lambda_1 = \{(a, 0.7), (b, 0.5), (c, 0.2)\}$, $\lambda_2 = \{(a, 0.5), (b, 0.4), (c, 1)\}$, $\lambda_3 = \{(a, 0.6), (b, 1), (c, 0.4)\}$ and $\lambda_4 = \{(a, 0.4), (b, 0.5), (c, 0.1)\}$. Let $\tau = \{0, \lambda_1, \lambda_2, 1\}$. The collection of fgspr-open sets $\lambda = \{0, \lambda_3, \lambda_4, 1\}$ such that $\lambda_2 \leq \vee\lambda$, then λ is fgspr-open cover of a fuzzy set λ_2 in X. Let the sub collection of fgspr-open sets $\mu = \{0, \lambda_3, 1\}$ such that $\lambda_2 \leq \vee\mu$. Therefore (X, τ) has a finite sub cover. Hence (X, τ) is fgspr compact.

Theorem 3.7: Let X be a fgspr compact space and λ be a fgspr-closed set in X. Then λ is fgspr compact.

Proof: Suppose λ be a fgspr-closed set in X. Let $\{\mu_i : i \in I\}$ be a fgspr-open cover of λ in X. So by fgspr-closedness of λ, $spcl(\lambda) \leq \vee\{\mu_i : i \in I\}$. Now spcl(λ) is a fgspr-closed set and hence it is fgspr compact. This implies that, there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $spcl(\lambda) \leq \vee_{j=1}^{n}\{\mu_{i_j} : i \in I\}$ and so $\lambda \leq \vee_{j=1}^{n}\{\mu_{i_j} : i \in I\}$. Therefore λ has finite sub cover and hence λ is fgspr compact.

Theorem 3.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is a fgspr-continuous function and surjective. If X is a fgspr compact space, then Y is a fgspr compact space.

Proof: Let $\{\lambda_i : i \in I\}$ be a fuzzy open cover of Y then $\vee\{\lambda_i : i \in I\} = 1$. Since every fuzzy open set is a fgspr-open set and f is fgspr-continuous, $\{f^{-1}(\lambda_i) : i \in I\}$ is a fgspr-open cover of X. Then $\vee\{f^{-1}(\lambda_i) : i \in I\} = 1$. Since X is a fgspr compact space, then there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $\vee_{j=1}^{n}\{f^{-1}(\lambda_{i_j}) : i \in I\} = 1$. (i.e) $f^{-1}[\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\}] = 1$. Since f is surjective, $f(f^{-1}[\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\}]) = f(1)$. (i.e) $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 1$. Hence Y is a fgspr compact space.

Theorem 3.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is a fgspr-irresolute function and surjective. If X is a fgspr compact space, then Y is a fgspr compact space.

Proof: Let $\{\lambda_i : i \in I\}$ be a fgspr-open cover of Y then $\vee\{\lambda_i : i \in I\} = 1$. Since f is fgspr-irresolute, $\{f^{-1}(\lambda_i) : i \in I\}$ is a fgspr-open cover of X. Then $\vee\{f^{-1}(\lambda_i) : i \in I\} = 1$. Since X is a fgspr compact space, then there exists a finite sub collection $\{i_1, i_2,\ldots,i_n \in I\}$ such that $\vee_{j=1}^{n}\{f^{-1}(\lambda_{i_j}) : i \in I\} = 1$. (i.e) $f^{-1}[\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\}] = 1$. Since f is surjective, $f(f^{-1}[\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\}]) = f(1)$. (i.e) $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 1$. Hence Y is a fgspr compact space.

Theorem 3.10: A fuzzy topological space X is a fgspr compact space if and only if for every collection $\{\lambda_i : i \in I\}$ of fgspr-closed sets of X having the finite intersection property $\wedge\{\lambda_i : i \in I\} \neq 0$.

Proof: Suppose X is a fgspr compact space and let $\{\lambda_i : i \in I\}$ be a collection of fgspr-closed sets of X with the finite intersection property. To show that $\wedge\{\lambda_i : i \in I\} \neq 0$. Suppose that $\wedge\{\lambda_i : i \in I\} = 0$, then $1 - \wedge\{\lambda_i : i \in I\} = 1$. This implies that, $\vee\{(1-\lambda_i) : i \in I\} = 1$. Since X is a fgspr compact space, then there exists a finite sub collection $\{i_1, i_2,\ldots,i_n \in I\}$ such that $\vee_{j=1}^{n}\{(1-\lambda_{i_j}) : i \in I\} = 1$. Then $\wedge_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 0$, which is a contradiction. Therefore $\wedge\{\lambda_i : i \in I\} \neq 0$.
Conversely, let $\{\lambda_i : i \in I\}$ be a fgspr-open cover of X then $\vee\{\lambda_i : i \in I\} = 1$. Suppose that for every finite sub collection $\{i_1, i_2,\ldots,i_n \in I\}$, we have $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} \neq 1$, then $\wedge_{j=1}^{n}\{(1-\lambda_{i_j}) : i \in I\} \neq 0$. Hence $\{(1-\lambda_i), i \in I\}$ satisfies the finite intersection property. Then from the hypothesis, we have $\wedge\{(1-\lambda_i) : i \in I\} \neq 0$. (i.e) $\vee\{\lambda_i : i \in I\} \neq 1$, which is a contradiction. Hence X is a fgspr compact space.

Theorem 3.11: Let f: $(X, \tau) \to (Y, \sigma)$ be a fgspr-continuous function. If λ is fuzzy compact relative to X, then $f(\lambda)$ is fgspr compact relative to Y.

Proof: Let the collection $\{\mu_i : i \in I\}$ be fuzzy open sets of $f(\lambda)$ in Y. (i.e) $f(\lambda) \leq \vee\{\mu_i : i \in I\}$. Since every fuzzy open set is a fgspr-open set and f is fgspr-continuous, then the collection $\{f^{-1}(\mu_i), i \in I\}$ is fgspr-open sets in X. (i.e) $f^{-1}(f(\lambda)) \leq f^{-1}(\vee\{\mu_i : i \in I\})$. (i.e) $\lambda \leq \vee\{f^{-1}(\mu_i) : i \in I\}$. Since λ is fuzzy compact relative to X, then there exists a finite sub collection $\{i_1, i_2,\ldots,i_n \in I\}$ such that $\lambda \leq \vee_{j=1}^{n}\{f^{-1}(\mu_{i_j}) : i \in I\}$. (i.e) $f(\lambda) \leq f[\vee_{j=1}^{n}\{f^{-1}(\mu_{i_j}) : i \in I\}] = \vee_{j=1}^{n}[f\{f^{-1}(\mu_{i_j}) : i \in I\}]$. (i.e) $f(\lambda) \leq \vee_{j=1}^{n}\{\mu_{i_j} : i \in I\}$. Hence $f(\lambda)$ is fgspr compact relative to Y.

Theorem 3.12: Let f: $(X, \tau) \to (Y, \sigma)$ be a fgspr-continuous function. If λ is fuzzy compact in X, then $f(\lambda)$ is fgspr compact in Y.

Proof: By Theorem 3.11, $f(\lambda)$ is fgspr compact relative to Y and by Definition 3.4, $f(\lambda)$ is fgspr compact in Y.

Theorem 3.13: Let f: $(X, \tau) \to (Y, \sigma)$ be a fgspr-irresolute function. If λ is fgspr compact relative to X, then $f(\lambda)$ is fgspr compact relative to Y.

Proof: Let the collection $\{\mu_i : i \in I\}$ be fgspr-open sets of $f(\lambda)$ in Y. (i.e) $f(\lambda) \leq \vee\{\mu_i : i \in I\}$. Since f is fgspr-irresolute, then the collection $\{f^{-1}(\mu_i) : i \in I\}$ is fgspr-open sets in X. (i.e) $f^{-1}(f(\lambda)) \leq f^{-1}(\vee\{\mu_i : i \in I\})$. (i.e) $\lambda \leq \vee\{f^{-1}(\mu_i) : i \in I\}$. Since λ is fgspr compact relative to X, then there exists a finite sub collection $\{i_1, i_2,\ldots,i_n \in I\}$ such that $\lambda \leq \vee_{j=1}^{n}\{f^{-1}(\mu_{i_j}) : i \in I\}$. (i.e) $f(\lambda) \leq f[\vee_{j=1}^{n}\{f^{-1}(\mu_{i_j}) : i \in I\}] = \vee_{j=1}^{n}[f\{f^{-1}(\mu_{i_j}) : i \in I\}]$. (i.e) $f(\lambda) \leq \vee_{j=1}^{n}\{\mu_{i_j} : i \in I\}$. Hence $f(\lambda)$ is fgspr compact relative to Y.

Theorem 3.14: Let f: $(X, \tau) \to (Y, \sigma)$ be a fgspr-irresolute function. If λ is fgspr compact in X, then $f(\lambda)$ is fgspr compact in Y.

Proof: By Theorem 3.13, $f(\lambda)$ is fgspr compact relative to Y and by Definition 3.4, $f(\lambda)$ is fgspr compact in Y.

Theorem 3.15: Let f: $(X, \tau) \to (Y, \sigma)$ be fgspr-open bijective and Y is a fgspr compact space, then X is a fgspr compact space.

Proof: Let $\{\lambda_i : i \in I\}$ be a fuzzy open cover of X then $\vee\{\lambda_i : i \in I\} = 1$. Since every fuzzy open set is a fgspr-open set and f is fgspr-open, $\{f(\lambda_i) : i \in I\}$ is a fgspr-open cover of Y. Then $\vee\{f(\lambda_i) : i \in I\} = 1$. Since Y is a fgspr compact space, then there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $\vee_{j=1}^{n}\{f(\lambda_{i_j}) : i \in I\} = 1$. (i.e) $f(\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\}) = 1$. Since f is bijective, $f^{-1}[f(\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\})] = f^{-1}(1)$. (i.e) $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 1$. Hence X is a fgspr compact space.

Theorem 3.16: Every fgspr compact space is a fuzzy compact space.

Proof: Let $\{\lambda_i : i \in I\}$ be a fuzzy open cover of X then $\vee\{\lambda_i : i \in I\} = 1$. Since every fuzzy open set is a fgspr-open set, then $\{\lambda_i : i \in I\}$ is a fgspr-open cover of X. Since X is a fgspr compact space, then there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 1$. Hence X is a fuzzy compact space.

Theorem 3.17: If X is fuzzy semi preregular $T_{1/2}^{*}$ space and X is a fuzzy compact space then X is a fgspr compact space.

Proof: Let $\{\lambda_i : i \in I\}$ be a fgspr-open cover of X then $\vee\{\lambda_i : i \in I\} = 1$. Since X is fuzzy semi preregular $T_{1/2}^{*}$ space, then $\{\lambda_i : i \in I\}$ is a fuzzy open cover of X. Since X is a fuzzy compact space, then there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 1$. Hence X is a fgspr compact space.

Theorem 3.18: Every fgspr compact space is a fsp-compact space.

Proof: Let $\{\lambda_i : i \in I\}$ be a fsp-open cover of X then $\vee\{\lambda_i : i \in I\} = 1$. Since every fuzzy semi preopen set is a fgspr-open set, then $\{\lambda_i : i \in I\}$ is a fgspr-open cover of X. Since X is a fgspr compact space, then there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 1$. Hence X is a fsp-compact space.

Theorem 3.19: If X is fuzzy semi preregular $T_{1/2}$ space and X is a fsp-compact space then X is a fgspr compact space.

Proof: Let $\{\lambda_i : i \in I\}$ be a fgspr-open cover of X then $\vee\{\lambda_i : i \in I\} = 1$. Since X is fuzzy semi preregular $T_{1/2}$ space, then $\{\lambda_i : i \in I\}$ is a fsp-open cover of X. Since X is a fsp-compact space, then there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $\vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} = 1$. Hence X is a fgspr compact space.

Proposition 3.20: Let (X, τ) be a fuzzy topological space. If A and B are two fgspr compact subsets of X, then $A \vee B$ is also fgspr compact.

Proof: Let the collection $\{\lambda_\alpha : \alpha \in I\}$ be fgspr-open sets of $A \vee B$, then $A \vee B \leq \vee\{\lambda_\alpha : \alpha \in I\}$. Since $A \leq A \vee B$ and $B \leq A \vee B$, then the collection $\{\lambda_\alpha : \alpha \in I\}$ is fgspr-open sets of A and fgspr-open sets of B. Since A and B are two fgspr compact subsets, then there exists a finite sub collection $\{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n \in I\}$ which covering A belong to $\{\lambda_\alpha : \alpha \in I\}$ such that $A \leq \vee_{i=1}^{n}\{\lambda_{\alpha_i} : \alpha \in I\}$ and there exists a finite sub collection $\{\alpha_1, \alpha_2, \ldots\ldots, \alpha_m \in I\}$ which covering B belong to $\{\lambda_\alpha : \alpha \in I\}$ such that $B \leq \vee_{j=1}^{m}\{\lambda_{\alpha_j} : \alpha \in I\}$. Hence there exists a finite sub collection $\{\alpha_1, \alpha_2, \ldots\ldots, \alpha_{n+m} \in I\}$ which covering $A \vee B$ belong to $\{\lambda_\alpha : \alpha \in I\}$ such that $A \vee B \leq \vee_{k=1}^{n+m}\{\lambda_{\alpha_k} : \alpha \in I\}$. Thus $A \vee B$ is fgspr compact.

Proposition 3.21: Let (X, τ) be a fuzzy topological space. If A and B are two fgspr compact subsets of X, then $A \wedge B$ need not be fgspr compact.

Proposition 3.22: Let A and B be fuzzy subsets of a fuzzy topological space (X, τ) such that A is fgspr compact and B is a fgspr-closed set in X, then $A \wedge B$ is fgspr compact.

Proof: Let A is fgspr compact and B is a fgspr-closed set in X. To prove that $A \wedge B$ is a fgspr compact set. Let $\{\lambda_i : i \in I\}$ be a fgspr-open cover of $A \wedge B$. Since $1 - B$ is a fgspr-open set, then $\{\lambda_i : i \in I\} \vee (1 - B)$ is a fgspr-open cover of A. Since A is fgspr compact, then there exists a finite sub collection $\{i_1, i_2, \ldots\ldots, i_n \in I\}$ such that $A \leq \vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} \vee (1 - B)$. Therefore $A \wedge B \leq \vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\}$. Hence $A \wedge B$ is fgspr compact.

Theorem 3.23: Every fgspr-closed subset of a fgspr compact space is fgspr compact relative to X.

Proof: Let X be a fgspr compact space and A be any fgspr-closed subset of X. Let $\{\lambda_i : i \in I\}$ be a fgspr-open cover of X. Since A is fgspr-closed, then 1 – A is fgspr-open and $\{\lambda_i : i \in I\} \vee (1 - A)$ is a fgspr-open cover of X. Since X is a fgspr compact space, then there exists a finite sub collection $\{i_1, i_2, \ldots, i_n \in I\}$ such that $X \leq \vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\} \vee (1 - A)$. Therefore $A \leq \vee_{j=1}^{n}\{\lambda_{i_j} : i \in I\}$. Hence A is fgspr compact relative to X.

Theorem 3.24: Every fgspr-closed subset of a fgspr compact space is fgspr compact.

Proof: By Theorem 3.23, A is fgspr compact relative to X and by Definition 3.4, A is fgspr compact.

Theorem 3.25: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be fgspr-open and injective. If λ is fuzzy compact relative to Y, then $f^{-1}(\lambda)$ is fgspr compact relative to X.

Proof: Let the collection $\{\mu_i : i \in I\}$ be fuzzy open sets of $f^{-1}(\lambda)$ in X. (i.e) $f^{-1}(\lambda) \leq \vee\{\mu_i : i \in I\}$. Since f is fgspr-open and injective, then the collection $\{f(\mu_i) : i \in I\}$ is fgspr-open sets in Y. (i.e) $f(f^{-1}(\lambda)) \leq f(\vee\{\mu_i : i \in I\})$. (i.e) $\lambda \leq \vee\{f(\mu_i) : i \in I\}$. Since λ is fuzzy compact relative to Y, then there exists a finite sub collection $\{i_1, i_2, \ldots, i_n \in I\}$ such that $\lambda \leq \vee_{j=1}^{n}\{f(\mu_{i_j}) : i \in I\}$. (i.e) $f^{-1}(\lambda) \leq f^{-1}[\vee_{j=1}^{n}\{f(\mu_{i_j}) : i \in I\}] = \vee_{j=1}^{n}\{f^{-1}f(\mu_{i_j}) : i \in I\}$. (i.e) $f^{-1}(\lambda) \leq \vee_{j=1}^{n}\{\mu_{i_j} : i \in I\}$. Hence $f^{-1}(\lambda)$ is fgspr compact relative to X.

Theorem 3.26: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be fgspr-open and injective. If λ is fuzzy compact in Y, then $f^{-1}(\lambda)$ is fgspr compact in X.

Proof: By Theorem 3.25, $f^{-1}(\lambda)$ is fgspr compact relative to X and by Definition 3.4, $f^{-1}(\lambda)$ is fgspr compact in X.

IV. FGSPR COMPACT FUNCTION

Definition 4.1: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy generalized semi preregular compact (briefly, fgspr compact) if the inverse image of every fuzzy compact set in Y is a fgspr compact set in X.

Theorem 4.2: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is fgspr compact and g: $(Y, \sigma) \rightarrow (Z, \eta)$ is fuzzy compact, then g o f: $(X, \tau) \rightarrow (Z, \eta)$ is fgspr compact.

Proof: Let λ be a fuzzy compact set in Z. Since g is fuzzy compact, then $g^{-1}(\lambda)$ is a fuzzy compact set in Y. Since f is fgspr compact, then $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is a fgspr compact set in X. Hence g o f: $(X, \tau) \rightarrow (Z, \eta)$ is fgspr compact.

Theorem 4.3: If g o f: $(X, \tau) \rightarrow (Z, \eta)$ is fuzzy compact, f: $(X, \tau) \rightarrow (Y, \sigma)$ is fgspr-open and onto, then g: $(Y, \sigma) \rightarrow (Z, \eta)$ is fgspr compact.

Proof: Let λ be a fuzzy compact set in Z. Since g o f is fuzzy compact, then $(g \circ f)^{-1}(\lambda)$ is a fuzzy compact set in X. Since f is fgspr-open, $f[(gof)^{-1}](\lambda)$ is a fgspr compact set in Y. Since f is onto, then $f[(gof)^{-1}](\lambda) = g^{-1}(\lambda)$ is a fgspr compact set in Y. Therefore g: $(Y, \sigma) \rightarrow (Z, \eta)$ is fgspr compact.

Theorem 4.4: If gof: $(X, \tau) \rightarrow (Z, \eta)$ is fgspr compact, g: $(Y, \sigma) \rightarrow (Z, \eta)$ is fuzzy open and one to one, then f: $(X, \tau) \rightarrow (Y, \sigma)$ is fgspr compact.

Proof: Let λ be a fuzzy compact set in Y. Since g is fuzzy open, then $g(\lambda)$ is a fuzzy compact set in Z. Since g o f is fgspr compact, then $(g \circ f)^{-1}[g(\lambda)]$ is a fgspr compact set in X. Since g is one to one, then $(g \circ f)^{-1}[g(\lambda)] = f^{-1}(\lambda)$. Hence $f^{-1}(\lambda)$ is a fgspr compact set in X. Therefore f: $(X, \tau) \rightarrow (Y, \sigma)$ is fgspr compact.

Proposition 4.5: For any fgspr-closed set λ of a fuzzy topological space X, the inclusion function $i_\lambda : \lambda \rightarrow (X, \tau)$ is fgspr compact.

Proof: Let μ be a fgspr compact set in X, then by Proposition 3.22, $\lambda \wedge \mu$ is a fgspr compact set in λ. But $i_\lambda^{-1}(\mu) = \lambda \wedge \mu$, then $i_\lambda^{-1}(\mu)$ is a fgspr compact set in λ. Therefore the inclusion function $i_\lambda : \lambda \rightarrow (X, \tau)$ is fgspr compact.

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